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A THREE-DIMENSIONAL CONSTITUTIVE THEORY FOR FIBER COMPOSITE LAMINATED MEDIA

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13. ABSTRACT *(Maximum 200 words)*

A three-dimensional elastic constitutive theory is developed for application to fiber composite laminated media. The lamina level constitutive relationship is a specific subset of general transversely isotropic media behavior. This special class of lamina behavior permits the development of an exact lamination procedure for systems assembled from a single lamina type. The three-dimensional constitutive form for the laminate is determined in terms of the sub-scale lamina properties and the orientations of each lamina. The extension of this specific constitutive relationship to general transversely isotropic lamina involves separation of the five lamina-scale properties into fiber-dominated vs. matrix dominated classifications and the development of a generalized averaging procedure for the matrix-dominated properties. The resulting three-dimensional constitutive/lamination theory is evaluated through comparisons between exact solutions, using data bases appropriate for graphite and glass epoxy systems in quasi-isotropic lay-ups. The theory remains highly effective through the transition from thin laminate to thick laminate behavior and even beyond that through the transition from thick laminate behavior to fully and strongly three-dimensional elastic behavior. The generalized averaging procedure for the matrix-dominated properties produces variations in results that are the same or less than the variations in results due to the experimental uncertainty in the matrix-dominated properties themselves. The theory is fairly simple and extremely versatile in its application.

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1. THE PROBLEM

The means and mechanism for translating lamina level properties into laminate properties has been available for many years under strongly limited conditions loosely referred to as those of a "thin" laminate. This procedure is known as classical lamination theory, and it neglects the interface conditions between individual lamina and directly sums, by algebraic formulas, the inplane lamina properties to obtain the integrated inplane laminate properties. This two-dimensional (2-D) procedure is widely used and has contributed greatly to the effectiveness of fiber composites. Unfortunately, the 2-D procedure provides little guidance on how to proceed in the much more complicated three-dimensional (3-D) case. However, Pagano (1974) has provided a complete and general 3-D lamination procedure, and it has been implemented by Sun and Li (1988). The 2-D, classical lamination procedure is approximate in the sense that it implies plane stress conditions. It is the intention here to develop a 3-D lamination procedure which retains the essential simplicity of the 2-D procedure. Of course, it will not be possible to assume plane stress conditions in the 3-D case, and some other approximations will need to be introduced here to accomplish the objective.

The starting point in the materials characterization is the properties state at the unidirectional lamina level. Taking these properties to be characterized by a state of transversely isotropic symmetry leads to the properties specification through five independent properties. These five independent elastic properties can be specified by the longitudinal Young's modulus, E_l, meaning the modulus in the fiber direction, the longitudinal Poisson's ratio, ν_l, meaning the transverse strain response when the composite is strained in the longitudinal direction to determine E_l, as well as the transverse Young's modulus, E_t, the longitudinal shear modulus, μ_l, and finally the transverse shear modulus, μ_t. These five lamina level properties can be differentiated by grouping them into fiber-dominated vs. matrix-dominated properties. The two grouping are:

Fiber Dominated	**Matrix Dominated**
E_l	E_t
ν_l	μ_l
	μ_t

For the fiber-dominated properties, the fibers act as the direct load transfer agent. In the matrix-dominated properties, the fibers effectively act as "inclusions" in a continuous matrix phase. This distinction is fundamental, and reveals itself in all credible micro-mechanics models of fiber composites. High performance fibers translate into a fiber-property dominated composite. Accordingly, in a fiber-dominated composite, the matrix-dominated properties are of lesser importance than are the fiber-dominated properties. Leaving the fiber dominated properties unchanged, we specifically seek to develop an averaging procedure for the less important three matrix-dominated properties. The procedure is intended to simplify the constitutive form, but it must not significantly alter basic physical behavior. Needless to say, the matrix dominated properties averaging procedure will be of no interest or use if it does not permit the development of a 3-D lamination theory and the subsequent detailed evaluation there of. Furthermore, it remains possible that the entire procedure could be "exact" in special cases, as will turn out to be true.

2. THE FORMAL METHOD

The macroscopic elastic properties for the aligned fiber reinforced medium are taken to be those of transversely isotropic symmetry with

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{22} & 0 & 0 & 0 \\ & & & (C_{22}-C_{23})/2 & 0 & 0 \\ & & & & C_{66} & 0 \\ & & & & & C_{66} \end{bmatrix} , \qquad (1)$$

where $\sigma_i = C_{ij}\varepsilon_j$ and with

$$\sigma_1 = \sigma_{11},\ \ \sigma_2 = \sigma_{22},\ \ \sigma_3 = \sigma_{33},\ \ \sigma_4 = \sigma_{23},\ \ \sigma_5 = \sigma_{31},\ \ \sigma_6 = \sigma_{12}$$

and

$$\varepsilon_1 = \varepsilon_{11},\ \ \varepsilon_2 = \varepsilon_{11},\ \ \varepsilon_3 = \varepsilon_{33},\ \ \varepsilon_4 = 2\varepsilon_{23},\ \ \varepsilon_5 = 2\varepsilon_{31},\ \ \varepsilon_6 = 2\varepsilon_{12}\ .$$

The five independent properties in (1), with axis 1 in the fiber direction, can be written in terms of the usual measurable properties, E_{11}, ν_{12}, E_{22}, μ_{12}, and μ_{23} through:

$$\begin{aligned} C_{11} &= E_{11} + 4\nu_{12}^2\kappa_{23}\ , \\ C_{12} &= 2\kappa_{23}\nu_{12}\ , \\ C_{22} &= \mu_{23} + \kappa_{23}\ , \\ C_{23} &= -\mu_{23} + \kappa_{23}\ , \\ C_{66} &= \mu_{12}\ . \end{aligned} \tag{2}$$

Here

$$\kappa_{23} = \frac{E_{22}}{4\,[1 - \nu_{12}^2 E_{22}/E_{11}] - E_{22}/\mu_{23}}\ , \tag{3}$$

with E_{11} and E_{22} being uni-axial moduli, ν_{12}, the axial Poisson's ratio, and μ_{12} and μ_{23} the shear moduli.

The grouping of five independent properties, E_{11}, ν_{12}, E_{22}, μ_{12}, and μ_{23} contains the three matrix-dominated moduli, E_{22}, μ_{12}, and μ_{23} which were discussed previously. To permit developing a formalism, two interrelations are taken between the three matrix dominated properties. After obtaining the relevant forms, consideration will be returned to material systems for which the two interrelationships do not apply, which is the case of primary interest.

Begin by taking the two shear moduli, μ_{12} and μ_{23}, as being equal, *i.e.*,

$$\mu_{12} = \mu_{23}\ . \tag{4}$$

The second relationship between the matrix-dominated moduli is taken as

$$\mu_{23} = \frac{(1-\nu_{12})\,E_{22}}{2\,(1-\nu_{12}^2\,E_{22}/E_{11})} \,. \tag{5}$$

(This form actually results from a corresponding relationship between the C_{ij} moduli, namely, $C_{12} = C_{23}$.) Examining modulus C_{11} in light of the two forms (4) and (5), C_{11} can be expressed as

$$C_{11} = E_{11} - E + \hat{C}_{11} \,, \tag{6}$$

where

$$E = \frac{(1-\nu_{12}^2)\,E_{22}}{1-\nu_{12}^2\,E_{22}/E_{11}} = 2\,(1+\nu_{12})\,\mu_{12} = 2\,(1+\nu_{12})\,\mu_{23} \tag{7}$$

and

$$\hat{C}_{11} = \frac{(1-\nu_{12})^2\,E_{22}}{(1-\nu_{12}^2\,E_{22}/E_{11})\,(1-2\nu_{12})} = \frac{2\,(1-\nu_{12})\,\mu_{12}}{(1-2\nu_{12})} = \frac{2\,(1-\nu_{12})\,\mu_{23}}{(1-2\nu_{12})} \,. \tag{8}$$

With $\hat{C}_{11}$ so defined, for the other C_{ij}'s take

$$\hat{C}_{ij} = C_{ij}, \quad \text{when} \quad i, j \neq 1, 1 \,. \tag{9}$$

Then the full vector of $\hat{C}_{ij}$'s subject to (4) and (5) can be written as

$$\begin{bmatrix} \dfrac{1-2\nu_{12}}{2(1-\nu_{12})}\,\hat{C}_{11} \\[2ex] \dfrac{1-2\nu_{12}}{2\nu_{12}}\,\hat{C}_{12} \\[2ex] \dfrac{1-2\nu_{12}}{2(1-\nu_{12})}\,\hat{C}_{22} \\[2ex] \dfrac{1-2\nu_{12}}{2\nu_{12}}\,\hat{C}_{23} \\[2ex] \hat{C}_{66} \end{bmatrix} = \frac{(1-\nu_{12})\,E_{22}}{2(1-\nu_{12}^2\,E_{22}/E_{11})} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \mu_{12} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \mu_{23} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \,. \tag{10}$$

The form (10) rigorously applies only when (4) and (5) are satisfied, but it does provide a guiding formalism in the more general case of interest here.

At this point, consideration is returned to fiber reinforced systems which do not satisfy the two interrelationships, (4) and (5), and accordingly accommodate five independent properties. For such systems, it is our intention to develop an averaging procedure for E_{22}, μ_{12}, and μ_{23} which depends explicitly on all three properties. Of course these moduli cannot be averaged directly since E_{22} is a uni-axial modulus, while the other two are shear moduli. From (10), however, a format is evident by which E_{22} can be compared with μ_{12} and μ_{23}. Define generalized moduli by:

$$\mathcal{L}_1 = \frac{(1-\nu_{12})\,E_{22}}{2\,(1-\nu_{12}^2\,E_{22}/E_{11})}\ ,$$

$$\mathcal{L}_2 = \mu_{12}\ ,$$

$$\mathcal{L}_3 = \mu_{23}\ . \tag{11}$$

An averaged, generalized modulus, $\mathcal{L}$, could be established from (11) by arbitrarily writing, $\mathcal{L} = (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3)/3$. However, a better and more physically meaningfully averaging procedure now will be developed.

To establish a base-line accessible case for physical interpretation, examine the special form of transverse isotropy which results when the two Poisson's ratios are taken as vanishing, *i.e.*,

$$\nu_{12} = 0 \quad \text{and } \nu_{23} = 0\ . \tag{12}$$

From (2) it immediately follows for $\nu_{12} = 0$ that $C_{12} = 0$. The identity $E_{22} = 2(1 + \nu_{23})\mu_{23}$ combined with (12) produces $E_{22} = 2\mu_{23}$. Using this latter result in (3) and then (2) results in $C_{23} = 0$. Finally, using $E_{22} = 2\mu_{23}$ again in (3) and (2) gives $C_{22} = E_{22}$, so that (12) with (2) yields $C_{11} = E_{11}$. Combining all these results into (1) gives it the diagonal form

$$
[\,C_{ij}\,] = \begin{bmatrix} E_{11} & 0 & 0 & 0 & 0 & 0 \\ & E_{22} & 0 & 0 & 0 & 0 \\ & & E_{22} & 0 & 0 & 0 \\ & & & \mu_{23} & 0 & 0 \\ & & & & \mu_{12} & 0 \\ & & & & & \mu_{12} \end{bmatrix} . \tag{13}
$$

Thus, (13), with $E_{22} = 2\mu_{23}$, is the form assumed by transverse isotropy for vanishing Poisson's ratios, (12), with no other restrictions.

Now form the strain energy density, U, associated with (13). This gives

$$
2U = (E_{11} - E_{22})\,\varepsilon_{11}^2 + E_{22}\varepsilon_{11}^2 + E_{22}\varepsilon_{22}^2 + E_{22}\varepsilon_{33}^2 + 2\mu_{12}\varepsilon_{12}^2 + 2\mu_{23}\varepsilon_{23}^2 + 2\mu_{12}\varepsilon_{31}^2 \, . \tag{14}
$$

The term $(E_{11} - E_{22})\,\varepsilon_{11}^2$ is the fiber-dominated effect, which obviously must vanish as the fiber reinforcement contribution goes to zero in an isotropic matrix phase. For the remaining matrix-dominated terms in (14) let

$$
\hat{U} = U_{\text{Matrix-Dominated}} = \frac{E_{22}}{2}\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2\right) + \mu_{23}\varepsilon_{23}^2 + \mu_{12}\left(\varepsilon_{12}^2 + \varepsilon_{31}^2\right) . \tag{15}
$$

In the spirit of considering fully 3-D deformation conditions take all strain components as $\varepsilon_{ij} = O(\delta)$, and using this in (15) gives

$$
\hat{U} \approx O\left(\left[3\,\frac{E_{22}}{2} + 2\mu_{12} + \mu_{23}\right]\delta^2\right). \tag{16}
$$

It is seen that the three matrix-dominated moduli contribute to the strain energy (16) in the proportions

$$\frac{E_{22}}{2} \;:\; \mu_{12} \;:\; \mu_{23} \qquad \text{as} \qquad 3 \;:\; 2 \;:\; 1\,. \tag{17}$$

Returning to the generalized moduli, (11), an acceptable generalized averaging procedure must recover the result just found. Specifically, for the generalized moduli in (11) (specialized to $\nu_{12} = \nu_{23} = 0$) to recover the results (16) and (17), it is necessary that the moduli be averaged with the weighting factors shown as follows:

$$\mathcal{L}_1 \;:\; \mathcal{L}_2 \;:\; \mathcal{L}_3 \qquad \text{as} \qquad 3 \;:\; 2 \;:\; 1\,. \tag{18}$$

Thus, this generalized averaging procedure gives the generalized shear modulus, μ, as

$$\mu = \frac{1}{6}\left[\frac{3(1-\nu_{12})\,E_{22}}{2\left(1-\nu_{12}^2\,E_{22}/E_{11}\right)} + 2\mu_{12} + \mu_{23}\right]. \tag{19}$$

Finally, then the $\hat{C}_{ij}$ coefficients formalized by (10) are now taken as

$$\begin{bmatrix} \dfrac{1-2\nu_{12}}{2(1-\nu_{12})} & \hat{C}_{11} \\[2ex] \dfrac{1-2\nu_{12}}{2\nu_{12}} & \hat{C}_{12} \\[2ex] \dfrac{1-2\nu_{12}}{2(1-\nu_{12})} & \hat{C}_{22} \\[2ex] \dfrac{1-2\nu_{12}}{2\nu_{12}} & \hat{C}_{23} \\[2ex] & \hat{C}_{66} \end{bmatrix} = \mu \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \tag{20}$$

where μ is now given by (19). Relation (20) also applies in the "exact" case when (4) and (5) are identically satisfied.

Next, write the $\hat{C}_{ij}$'s in (20) in matrix form and compare them with the appropriate isotropic material form in Green and Zerna (1963). It is quickly seen that the forms in (20) are those for isotropic behavior with isotropic shear modulus given by μ and with isotropic λ given by

$$\lambda = \frac{2\nu_{12}}{1 - 2\nu_{12}}\mu . \tag{21}$$

The relation between $\hat{C}_{ij}$ and C_{ij} is given by (6) and (9). The $\hat{C}_{ij}$ form is isotropic, as just established. Thus, using (9), the C_{ij} form is isotropic to within the presence of the $(E_{11} - E)$ part of C_{11}. It directly follows that the stress constitutive relation, corresponding to the C_{ij} form, is given by

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} + (E_{11} - E)\delta_{1i}\delta_{1j}\varepsilon_{11} , \tag{22}$$

where λ is given by (21), μ by (19) and E is given by

$$E = 2(1 + \nu_{12})\mu . \tag{23}$$

The form (22) was previously identified by Christensen (1988); however, in that work the associated interrelationships (4) and (5) were taken as restrictions, rather than being developed into the generalized averaging procedure given here and culminating in expression (19). Also in the work of Christensen (1988), emphasis was given to using (22) to develop an associated failure criterion. Attention here is given to using the form (22) to identify a specific 3-D lamination procedure and to evaluate it.

The lamina level constitutive formulation is now complete with the forms (19) and (21-23). It should be emphasized that the lamina constitutive equation (22) does not imply the interrelations (4) and (5), although it only is "exact" when they are satisfied. In the most general case, all five transversely isotropic properties enter the constitutive form (22). The

two fiber-dominated properties enter the constitutive forms directly, while all three matrix-dominated properties enter through the generalized averaging procedure result (19). Using the form (22) to specifying the lamina-level behavior, the corresponding laminated medium characterization will be now found. This is the so called 3-D lamination theory or procedure.

Express constitutive relation (22) in coordinates other than in the fiber direction. Take the angle of rotation of the coordinate system as θ from the fiber direction, rotated about axis x_3. With a_{ij} being the direction cosines matrix, the constitutive relation (22) takes the rotated form

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} + (E_{11} - E) a_{1i} a_{1j} a_{1k} a_{1l} \varepsilon_{kl} \,, \tag{24}$$

where

$$[a_{1i}] = [\cos\theta, \quad -\sin\theta, \quad 0] \,, \tag{25}$$

with (25) being the first row of a_{ij}. Now take N similar lamina in bonded contact, each with its individual direction, θ_n. The 3-D constitutive form (24) for each lamina is directly combined to give the 3-D laminated medium constitutive form

$$\sigma_{ij} = \lambda \varepsilon_{kk} \, \delta_{ij} + 2\mu \varepsilon_{ij} + \frac{(E_{11} - E)}{N} \sum_{n=1}^{N} m_i^{(n)} m_j^{(n)} m_k^{(n)} m_l^{(n)} \varepsilon_{kl} \,, \tag{26}$$

wherein from (25)

$$\left[m_i^{(n)} \right] = \left[\cos\theta_n, \quad -\sin\theta_n, \quad 0 \right] . \tag{27}$$

Relation (26) along with (19), (21), (23), and (27) constitute the 3-D constitutive theory for fiber composite laminated media assembled from a single lamina type, and involves all five independent properties.

It is readily verified from (26) that the interlaminar stress components σ_{33}, σ_{32}, and σ_{31} are continuous across the interfaces between lamina because the last term in (26) vanishes for these three stress components. By definition the displacements are continuous across the

interfaces since the entire constitutive form (26) is expressed in terms of the single strain tensor, ε_{ij}. Thus, the laminated medium constitutive form (26), as derived from the lamina constitutive form (22), is an exact 3-D result. No approximations are involved in satisfying interface conditions. It is implicit in obtaining (26) from (22) that the strain gradients are small over the number of lamina of interest, N. This condition will be generalized (relaxed) in the section on utilization. Although form (26) takes all lamina to be of equal thickness, it is easily extended by including thickness related weighting functions in it.

The constitutive form (26) for the laminated medium is extraordinarily compact and easy to use. If the two interrelations (4) and (5) are satisfied (to within experimental accuracy) by a given set of five transversely isotropic properties, then the entire procedure up through and including the final constitutive result, (26), is "exact" within the theory of linear elasticity. In this case, the results are identical to the C_{ij} matrix in Pagano (1974). If the two interrelations (4) and (5) are not satisfied, then the result, (26), is an approximation based upon the generalized averaging procedure for the three matrix-dominated properties. Under this condition it will be necessary to carefully evaluate the nature of the approximation as will be done in the next two sections.

3. A TEST CASE

Judging the total accuracy of the proposed constitutive relationship against the exact form necessitates comparing more than just the resulting effective lamina moduli. Although direct comparisons between individual moduli show the differences expected from "uni-axial" load states, they do not accurately capture the coupling between components which occurs naturally under a general 3-D load state. This section formulates a specific BV problem which first allows direct comparisons of various field quantities obtained by using both the exact and proposed constitutive relationships, and second shows the finite regimes where classical long-wavelength (LW) theory (essentially plane stress or classical flat lamination theories) is appropriate. To aid the reader in distinguishing variables associated with the constitutive formulation from those associated with the BV problem, direction notation and an $X-Y-Z$ coordinate frame are employed in defining and solving the test problem.

In adopting a test case for the 3-D lamination theory, it is necessary to select a particular lay-up pattern. There are two limiting cases for lay-up patterns. At one extreme is the degenerate case of all lamina being aligned, thus the laminate retains aligned-axis character. At the other extreme is a quasi-isotropic lay-up involving the most dispersed pattern of directions. The aligned laminate is a trivial application of lamination theory, since there are no true interfaces. The quasi-isotropic lay-up is in fact, the most extreme case, and provides the most severe test of a lamination theory. It requires the highest degree of interactive coupling between all lamina properties to produce laminate behavior.

3.1 The Boundary Value Problem. The problem considered herein is a "homogenized" solid lying parallel to the $X-Z$ plane. The laminated solid consists of many identical transversely-isotropic laminae oriented in a repetitive [0°, ±60°] order and is considered to be quasi-isotropic in the $X-Z$ plane. A sufficient number of laminae exist through the thickness, in the Y direction, such that the macroscopic composite properties are well represented by those of a homogenized single [0°, ±60°] sublaminate.

Without loss of generality, the coordinate positions X, Y, and Z are non-dimensionalized to the coordinates x, y, and z by half the thickness of the solid, $h/2$, (*i.e.*, $x = 2X/h$, $y = 2Y/h$, and $z = 2Z/h$) so that the upper and lower surfaces lie in the planes $y = 1$ and $y = -1$, respectively. Displacements in the z direction, as well as all derivatives taken with respect to z, are taken to vanish identically (yielding plane strain conditions). Imposed on the lower surface, $y = -1$, are traction-free boundary conditions; while on the upper surface, $y = 1$, a sinusoidal normal traction is applied which has the form

$$t_n = -P \sin(\alpha x) . \tag{28}$$

Here

$$\alpha = \frac{h\pi}{2L} , \tag{29}$$

P, the load magnitude, has units of stress, $2L$ is the loading wavelength, and the convention that positive normal tractions produce normal tensile stresses is employed. The solid extends "infinitely" in the x direction, but only the region from $x = 0$ to $x = 2L/h$ need be examined, due to periodicity. Furthermore, all body forces are identically zero. This problem has been

chosen because by varying α, essentially the ratio of h to L, many different loading conditions can be achieved. For values of $h/L << 1$, classical LW conditions result, and, as h/L increases, fully 3-D load states develop.

3.2 Material Formulation. Using the homogenizing procedure used by Sun and Li (1988), the effective sublaminate stiffnesses are assembled. Using the appropriate tensor transformations, the ±60° material stiffness tensor components are expressed in the 0° lamina material coordinate frame. (For transversely isotropic lamina whose plane of isotropy lies perpendicular to the lamina- or ply- plane, Christensen (1988) gives the explicit stiffness tensor components in any cartesian coordinate frame where the rotated inplane axes remain parallel to the original ply-plane.) Unit macro strains are individually imposed along the boundaries of a combined (unit-square) ±60° lamina pair. After enforcing continuity of interlaminar displacements and tractions, the resulting net forces yield the effective ±60° stiffness components when properly normalized. Next, the 0° and effective ±60° laminae are weighted appropriately and assembled using the same method to produce the effective properties of the entire sublaminate.

The quasi-isotropic lay-up produces a set of transversely isotropic properties in three dimensions. With x_3 being the axis of symmetry for the laminate, the non-zero effective stiffness properties, c_{ij}, for the [0°, ±60°] quasi-isotropic laminate are given by these newly derived (exact) closed form expressions:

$$c_{11} = c_{22} = \frac{3}{8}(C_{11} + C_{22}) + \frac{1}{4}C_{12} + \frac{1}{2}C_{66} - \frac{1}{8C_{22}}(C_{12} - C_{23})^2 , \tag{30}$$

$$c_{12} = \frac{1}{8}(C_{11} + C_{22}) + \frac{3}{4}C_{12} - \frac{1}{2}C_{66} + \frac{1}{8C_{22}}(C_{12} - C_{23})^2 , \tag{31}$$

$$c_{13} = c_{23} = \frac{1}{2}(C_{12} + C_{23}) , \tag{32}$$

$$c_{33} = C_{22} , \tag{33}$$

$$c_{44} = c_{55} = \frac{2C_{66}(C_{22} - C_{23})}{2C_{66} + C_{22} - C_{23}} , \tag{34}$$

$$c_{66} = \frac{1}{2}(c_{11} - c_{12}) . \tag{35}$$

Here the upper case C_{ij} refers to the individual lamina components given in the lamina coordinate frame; *i.e.*, axis 1 in the fiber direction. Substituting (2) and (3) into (30-35) allows the quasi-isotropic stiffnesses to be expressed in terms of the usual five properties, specifically E_{11}, E_{22}, ν_{12}, μ_{12}, and μ_{23}.

3.3 Solution Formulation. The general solution to the test-case BV problem has been fully derived, following the method employed by Timoshenko and Goodier (1970) to solve a similar isotropic problem. First, the equilibrium equations, infinitesimal strain-displacement relationships, and compatibility equations are reduced by incorporating the plane strain assumption. Next, the individual stress component forms are found by using the reduced equilibrium equations and by assuming a separable solution where a Fourier series represents the x dependence and some function of $F(y)$ represents the y dependence. The strain component forms, determined by substituting the stress forms into the constitutive relationship, along with the sole remaining compatibility equation yield the governing differential equations for $F(y)$. Solving the linear homogeneous differential equation and matching the original boundary conditions uniquely determines $F(y)$ which, when substituted back into the original stress and strain forms, yields the complete closed-form BV problem solution.

The effective laminate properties and BV problem, in conjunction with actual material properties, allow an objective and realistic evaluation of the proposed constitutive relationship in a fully 3-D load state. In the following section, laminate properties and solutions to the BV problem will be compared for two specific composite systems, and the effectiveness of the proposed lamination procedure relationships will be demonstrated.

4. EVALUATION

4.1 Properties. To evaluate the proposed constitutive relationship, two sets of lamina properties are chosen which approximately satisfy the restrictions of transverse isotropy. A graphite/epoxy (Gr/Ep) composite [AS4/3501-6] is chosen for its extremely anisotropic (axial to transverse) moduli, along with a glass/epoxy (Gl/Ep) composite, chosen as representative of a lesser fiber-dominated system. Table 1 lists the elastic lamina constants for both systems,

Table 1. Lamina Properties Used in Evaluation

	Gr/Ep †	Gl/Ep ††
E_{11} GPa	144	53.7
$E_{22} = E_{33}$ GPa	9.65	17.9
ν_{12}	0.31	0.25
ν_{23}	0.52	0.25
μ_{12} GPa	5.24	8.96
μ_{23} GPa	3.17	7.17

† Data from Kim, Abrams, and Knight, 1988.
†† Data from Sun and Li, 1988.

and to satisfy the restrictions of transverse isotropy, namely $\mu_{23} = E_{22}/2(1 + \nu_{23})$, the actual moduli are adjusted slightly from the reported values.

Table 2 contains the effective quasi-isotropic laminate properties calculated via (30-35) using the proposed forms from Section 2, namely the C_{ij} properties corresponding to the lamina constitutive equation (22), and, for comparison, using the exact C_{ij} properties from (2) and (3). [The proposed C_{ij} properties are explicitly given by (6), (9), (19-21) and (23).] The ratios of these results are shown in Table 2 as well. Generally, less discrepancy exists between the inplane moduli (1 and 2 directions) than the out-of-plane components. Also tabulated in Table 2 are the quantities $E_{11}/(1 - \nu_{12}^2)$ and $E_{11}/(1 - \nu_{12})$ which represent uni-axial and bi-axial inplane extensional stiffnesses, respectively, associated with classical flat lamination theory. For the two composite systems examined, the exact and proposed quasi-isotropic moduli and inplane extensional stiffnesses differ typically by less than ≈ 10%.

4.2 Test BV Problem. The previously formulated BV problem is now explored, utilizing the Gr/Ep and Gl/Ep laminate properties listed in Table 2, to quantify results from a 3-D problem. By judiciously selecting several variables at two locations, a meaningful and representative comparison is possible without having to examine the entire field. The variables chosen for comparison are: first the flexural stress and strain, $\sigma_{xx}(y = 1, x = L/h)$

Table 2. Effective Quasi-Isotropic Laminate Elastic Constants and Inplane Extensional Stiffnesses Calculated Using the Exact Transversely Isotropic and Proposed Lamina Constitutive Relationships for Gr/Ep and Gl/Ep Systems

	Gr/Ep			Gl/Ep		
	Proposed	Exact	Proposed/Exact	Proposed	Exact	Proposed/Exact
$E_{11} = E_{22}$ GPa	55.0	55.5	0.99	30.7	31.1	0.99
E_{33} GPa	13.4	12.5	1.07	20.4	18.3	1.11
ν_{12}	0.329	0.309	1.06	0.283	0.245	1.16
$\nu_{13} = \nu_{23}$	0.302	0.348	0.87	0.239	0.225	1.06
μ_{12} GPa	20.7	21.2	0.98	11.9	12.5	0.95
$\mu_{13} = \mu_{23}$ GPa	3.95	3.95	1.00	7.61	8.00	0.95
$E_{11}/(1-\nu_{12}^2)$ GPa	61.7	61.3	1.01	33.4	33.1	1.01
$E_{11}/(1-\nu_{12})$ GPa	82.0	80.0	1.02	42.8	41.1	1.04

and $\varepsilon_{xx}(y = 0, x = L/h)$; second the upper-surface normal displacement, $u_y(y = 1, x = L/h)$; and third the mid-plane shear stress and shear strain, $\sigma_{xy}(y = 0; x = L/h)$ and $\varepsilon_{xy}(y = 0, x = L/h)$. At these locations, these variables are the dominant components present as well as the minimum or maximum values achieved by these variables (almost) anywhere in the body. The normal displacement probably best captures the 3-D aspects in that it reflects an integrated value, where as the other terms are more indicative of the material properties directly associated with that particular component.

Table 3 lists the normalized flexural stresses, as a function of the thickness-to-length ratio, h/L, evaluated using both the proposed and exact constitutive relationships for both composite systems. The stresses have been normalized by the asymptotic solution obtained as $h/L \rightarrow 0$, *i.e.*, essentially LW or classical plate theory, using the exact transversely-isotropic lamina relationship. Reasonable agreement exists between the two solutions from $h/L = 0$ to an astonishing value of $h/L = 10$. Note how rapidly the BV solution deviates from classical LW theory as h/L increases beyond approximately 0.10. Figures 1 and 2 show the mid-plane shear strain and flexural strain, respectively, normalized by their LW solution for both

Table 3. Normalized Flexural Stresses for Gr/Ep and Gl/Ep Systems Verses Thickness to Length Ratio *h/L*

h/L	Gr/Ep		Gl/Ep	
	Proposed	Exact	Proposed	Exact
0.01	1.000	1.000	1.000	1.000
0.05	1.006	1.006	1.001	1.001
0.10	1.023	1.023	1.006	1.006
0.50	1.663	1.638	1.221	1.216
1.00	3.824	3.965	2.319	2.406
5.00	86.18	90.39	51.60	54.78
10.0	343.1	359.8	205.4	218.1

Note: Normalized flexural stresses are defined by $\sigma_{xx}(y = 1, x = L/h)/\sigma_{xx\text{-}LW}(y = 1, x = L/h)$, where the normalizing values, $\sigma_{xx\text{-}LW}$, represent the asymptotic solution obtained as $h/L \rightarrow 0$; *i.e.*, LW theory.

composite systems as a function of *h/L*. The agreement between the results shown in Figures 1 and 2 is typical of all the variables considered. Examining the figures reveals that the components calculated from the proposed and exact constitutive relationship differ only slightly even as $\varepsilon_{xy} \rightarrow 0$ as $h/L \rightarrow \infty$. Unfortunately, plotted at this scale it is not evident how drastically the BV and LW solutions drift apart, *e.g.*, $\varepsilon_{xx} > 2\varepsilon_{xx\text{-}LW}$ for $h/L > 0.75$ and $\varepsilon_{xy} < 0.75\varepsilon_{xy\text{-}LW}$ for $h/L > 1.0$. Nonetheless, the proposed lamina relationship captures remarkably well all characteristics of the BV problem, neglecting small differences in numerical values.

To quantify precisely the differences arising from the proposed relationship, the five field quantities have been normalized by their corresponding exact values and plotted verses *h/L* in Figures 3 and 4 for the Gr/Ep and Gl/Ep systems, respectively. Recognize that as $h/L \rightarrow \infty$, $\sigma_{xy} \rightarrow 0$ and $\varepsilon_{yy} \rightarrow 0$, and thus their normalized ratios for increasing *h/L* represent differences in decreasingly smaller numbers. Ignoring the regions where σ_{xy} and ε_{xy} approach zero, the errors in the five normalized field quantities for $h/L < 10$ are less than 11% in the Gl/Ep system and 7% in the fiber-dominated Gr/Ep system.

Overall, the BV problem results invariably reflect the differences in effective lamina properties between the proposed and exact constitutive idealization. Keeping in mind that the BV problem is intrinsically stress controlled, underestimated properties, like Gl/Ep μ_{13}, produce overestimated strain values and vice versa.

This test problem is ideally suited to discriminate thin laminate conditions (LW theory) from thick laminate conditions. The ratio of laminate thickness to the loading wavelength (*h/L*) directly gives the parameter of variation. As $h/L \rightarrow 0$, long wavelength, thin laminate conditions are recovered as a limiting case. As *h/L* becomes larger, more complex physical effects enter the BV problem. Somewhere in the range $0.1 < h/L < 1$ a transition from thin laminate conditions to thick laminate conditions occurs, *e.g.*, see Figures 1 and 2. Certainly, in the neighborhood of $h/L = 1$ fully 3-D deformation conditions exist in the laminate. The lamination theory developed herein properly models all major effects, and it even remains effective up through $h/L = 10$.

4.3 Constitutive Error Relative to Experimental Uncertainties. As stated in Section 2, this work aims to formulate a practical and workable 3-D lamination theory which retains algebraic simplicity while providing "reasonable and useful" composite representation. In actual application both constitutive approximations and material property variations, arising from experimental uncertainties, contribute to overall analysis inaccuracies. Thus, comparing the absolute mathematical error from the constitutive approximations with that from the material properties appears appropriate. This section compares the magnitude of experimental uncertainties to the resulting constitutive error in quantifying the lamina properties.

Large scatter in measured lamina properties is common, especially with all the difficulties encountered in obtaining them. For example, Kim, Abrams, and Knight (1988) report coefficients of variations in measured uni-directional Gr/Ep lamina moduli between 2.0% and 12.2% even though 3 to 10 specimens were used in each test. Clearly the lamina constants most difficult to experimentally determine and most prone to uncertainties are the three matrix-dominated moduli, namely, E_{22}, μ_{12}, and μ_{23}.

The differences between the proposed and exact lamina relationships stem solely from assumptions made regarding the matrix-dominated properties. Since variations in fiber-

dominated properties produce (nearly) identical results in either constitutive relationship, attention is focused only upon uncertainties associated with the matrix-dominated properties. Plotted in Figures 5 and 6 are the normalized BV problem quantities for the Gr/Ep and Gl/Ep systems, respectively, calculated by decreasing the three matrix-dominated lamina properties by 10% (an amount typical of their actual variation) from their base values given in Table 1. As before, the BV problem values are normalized by their corresponding exact quantities. The variations in field quantities and, thus, lamina properties caused by uncertainties in the three matrix-dominated properties are the same order of magnitude as those introduced by the proposed constitutive relationship; *i.e.*, compare Figures 4 and 5 with Figures 2 and 3. Therefore, we conclude that use of the proposed method in actual engineering analyses should produce results with uncertainties which, for all practical purposes, cannot be realistically differentiated from inherent experimental error in the matrix-dominated lamina properties.

5. UTILIZATION

This section explores how the proposed constitutive relationship can be beneficially utilized in analyzing both thin and thick composite structures.

5.1 Classical Lamination Theory. The proposed lamina relationship can be incorporated into classical lamination theory. Using the previous averaging approach on the four constants required by classical lamination theory, the generalized shear modulus becomes

$$\mu = \frac{1}{5}\left[\frac{3(1-\nu_{12})E_{22}}{2(1-\nu_{12}^2 E_{22}/E_{11})} + 2\mu_{12}\right]. \tag{36}$$

Using (6), (9), (20), and (36), the transformed n-th lamina stiffness matrix Q_{ij}^n, defined such that

$$
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}^n = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{22} & Q_{66} \end{bmatrix}^n \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}^n , \tag{37}
$$

is directly obtainable. Because of the lamina relationship form, Q_{ij}^n is decomposable into an effectively isotropic and homogeneous term, Q_{ij-H}^n, and a reinforcement term, Q_{ij-R}^n, as

$$
Q_{ij}^n = Q_{ij-H}^n + Q_{ij-R}^n , \tag{38}
$$

where

$$
[Q_{ij-H}]^n = \begin{bmatrix} 2\mu + \lambda & \lambda & 0 \\ \lambda & 2\mu + \lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} , \tag{39}
$$

and

$$
[Q_{ij-R}]^n = (E_{11} - E) \begin{bmatrix} \cos^4\theta_n & \cos^2\theta_n \sin^2\theta_n & -\cos^3\theta_n \sin\theta_n \\ \cos^2\theta_n \sin^2\theta_n & \sin^4\theta_n & -\cos\theta_n \sin^3\theta_n \\ -\cos^3\theta_n \sin\theta_n & -\cos\theta_n \sin^3\theta_n & \cos^2\theta_n \sin^2\theta_n \end{bmatrix} . \tag{40}
$$

Since μ_{23} does not enter into classical lamination theory, the form of μ given in (36) should be used. When evaluating the extensional, coupling, and bending stiffness matrices for a laminate composed of a single lamina type, the contribution of (39) factors through the integration, as a homogeneous isotropic material, leaving only the reinforcement contribution to be calculated on a ply by ply basis.

5.2 Finite Element Procedures for Thick Composite Structures. Currently, to analyze a thick composite structure either each lamina in the component is discretized and assigned its own material constants or the laminae are homogenized, represented by a single or several sets of effective properties, allowing the component to be discretized as an equivalent "homogeneous" solid. When defining each lamina as its own continuum layer, obtaining a satisfactory solution with modest to large through thickness gradients generally requires several elements through each lamina thickness; and because typical composite structures contain tens to hundreds of plies and because element aspect ratios must be maintained, the total problem size, *i.e.*, degrees of freedom (DOF), quickly escalates. In structures where adjacent parts mandate non-linear, presumably iterative, solution procedures and contain thousands to hundreds of thousands of DOF, implicit FE methods become economically unfeasible. On the other hand, when dynamic responses necessitate explicit FE schemes, *e.g.*, impact loadings, the maximum time increment permitted is limited by approximately the minimum time required for a wave to propagate across the smallest element (Bathe 1982). With such small elements, an unacceptable large number of time steps results and may render the solution technique impractical.

To circumvent some of these difficulties, consider incorporating the lamina constitutive form (24) into displacement-based solid elements, using conventional FE methodologies. Following the usual procedures, the laminated structure to be analyzed would be discretized as a homogeneous solid (presuming it is manufactured from a single composite system). However, when evaluating the elemental stiffness matrix K_{ij}, C_{ij} is allowed to vary with position representing the differently oriented lamina within the element. Symbolically, K_{ij} is thus calculated as

$$K_{ij} = \int_v B_{li}\, C_{lm}(x,y,z)\, B_{mj}\, dv\ , \tag{41}$$

where v designates the element volume, $C_{ij}(x, y,z)$ is the position dependent stiffness matrix whose form is given by (24), and B_{ij} is the strain-displacement transformation matrix. Note that (41) homogenizes the laminate material within each element to the same kinematic order as the element, without sacrificing stacking sequence related behavior. This procedure guarantees continuity of interlaminar displacements and tractions while ensuring that both the global solution and homogenization method converge, in the FE sense, as the number of elements increases. Simplifications in calculating (41) can be made by requiring the elements be oriented so that one surface is parallel to the plane of the lamina. Although the resulting strains would be the individual lamina strains, some post-processing is necessary to recover the inplane lamina stresses. In general, this approach should make analyzing thick laminates involve nearly the same degree of (analysis) difficulties as encountered in analyzing thick homogeneous solids.

6. CONCLUSION

A 3-D constitutive theory, which rigorously enforces continuity of interlaminar tractions and displacements, is developed for thick laminated media and is evaluated by direct comparisons with exact solutions. The lamina constitutive relationship, almost isotropic in mathematical form but arbitrarily anisotropic in physical content, simplifies lamination theory without sacrificing physical meaning or even significant accuracy. The accuracy level of this theory is well inside the range of experimental uncertainty contained in the properties themselves.

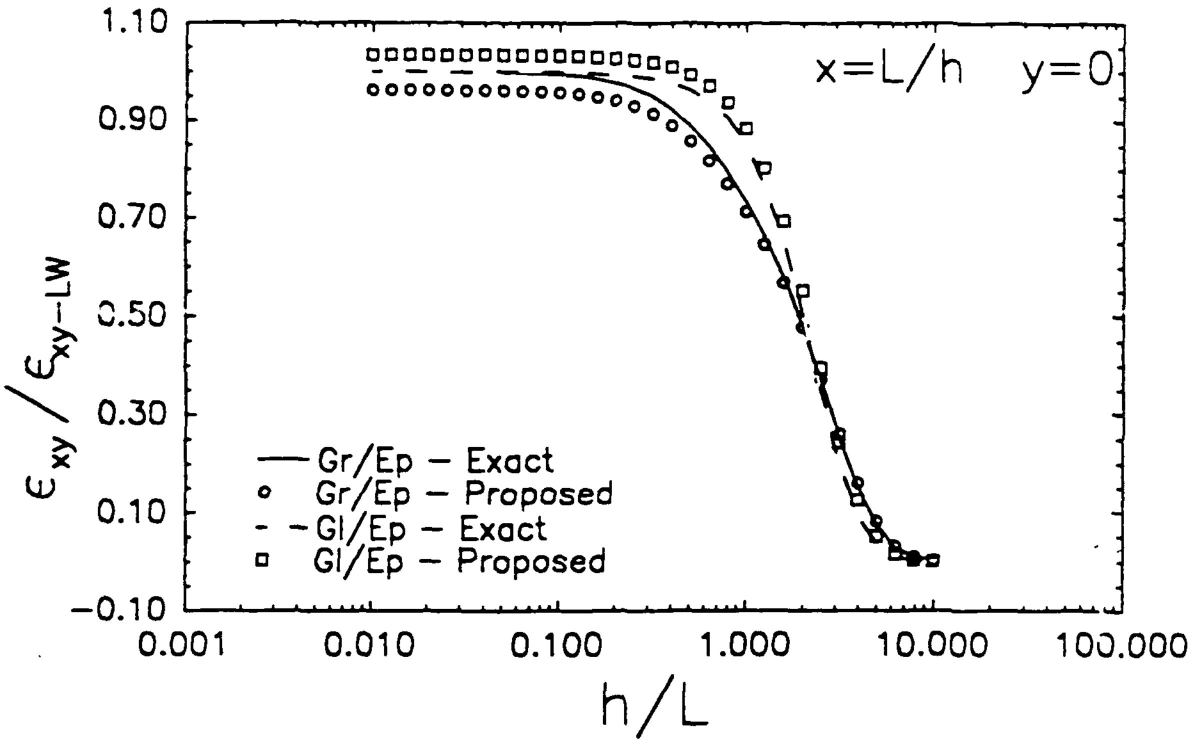

Figure 1. Mid-Plane Shear Strains, Normalized by the Exact Asymptotic Solution, vs. Thickness to Loading Wavelength Ratio for a Quasi-Isotropic Laminate.

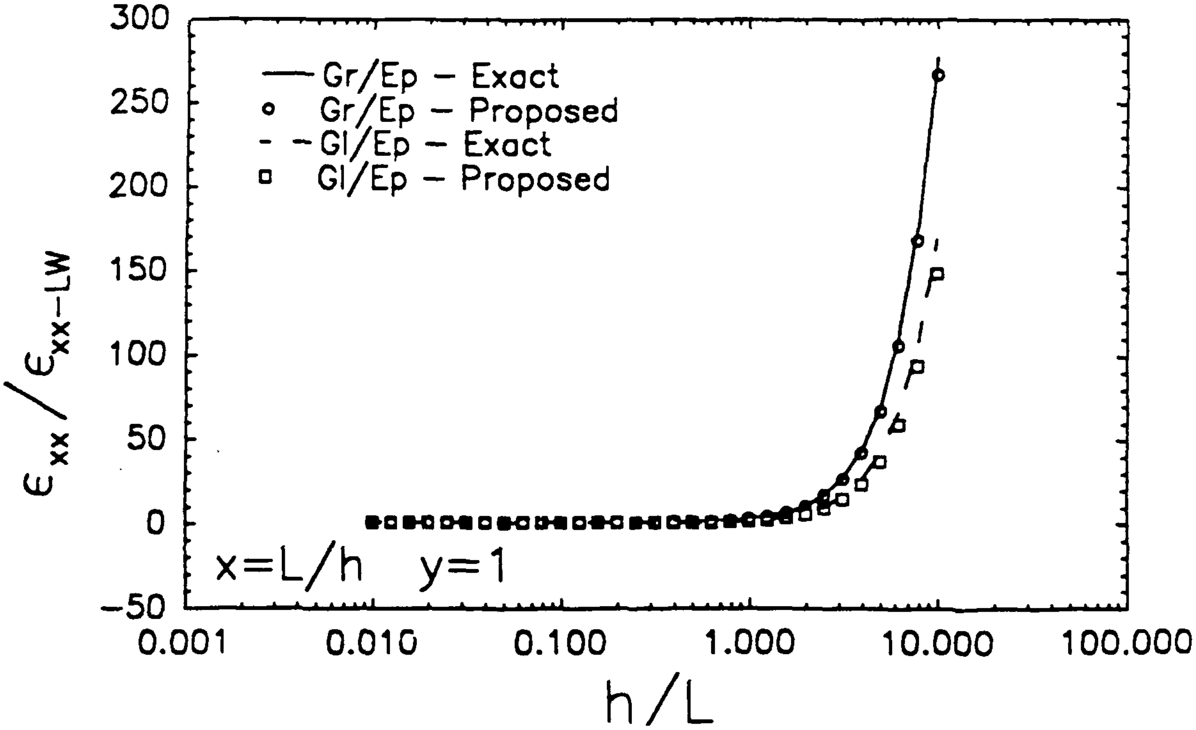

Figure 2. Flexural Strains, Normalized by the Exact Asymptotic Solution, vs. Thickness to Loading Wavelength Ratio for a Quasi-Isotropic Laminate.

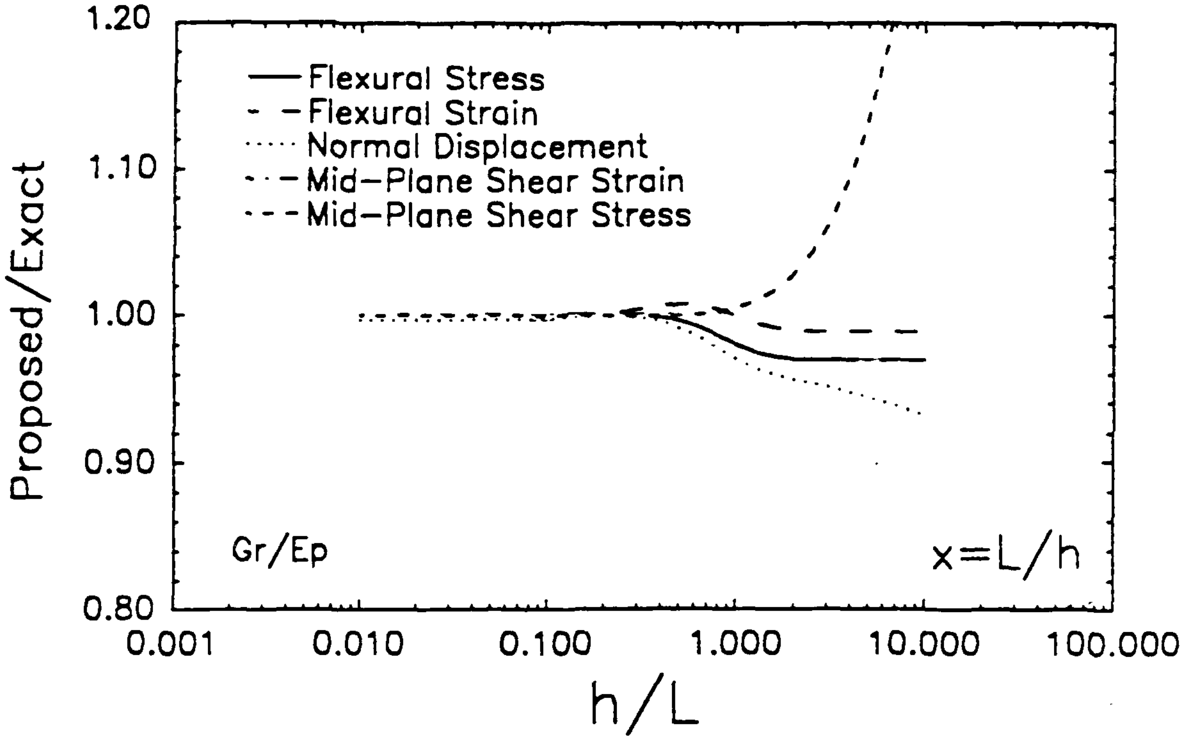

Figure 3. Normalized Field Quantities, as a Function of the Thickness to Loading Wavelength Ratio, Calculated Using the Proposed Constitutive Relationship and Normalized by Their Corresponding Exact Values, for Gr/Ep.

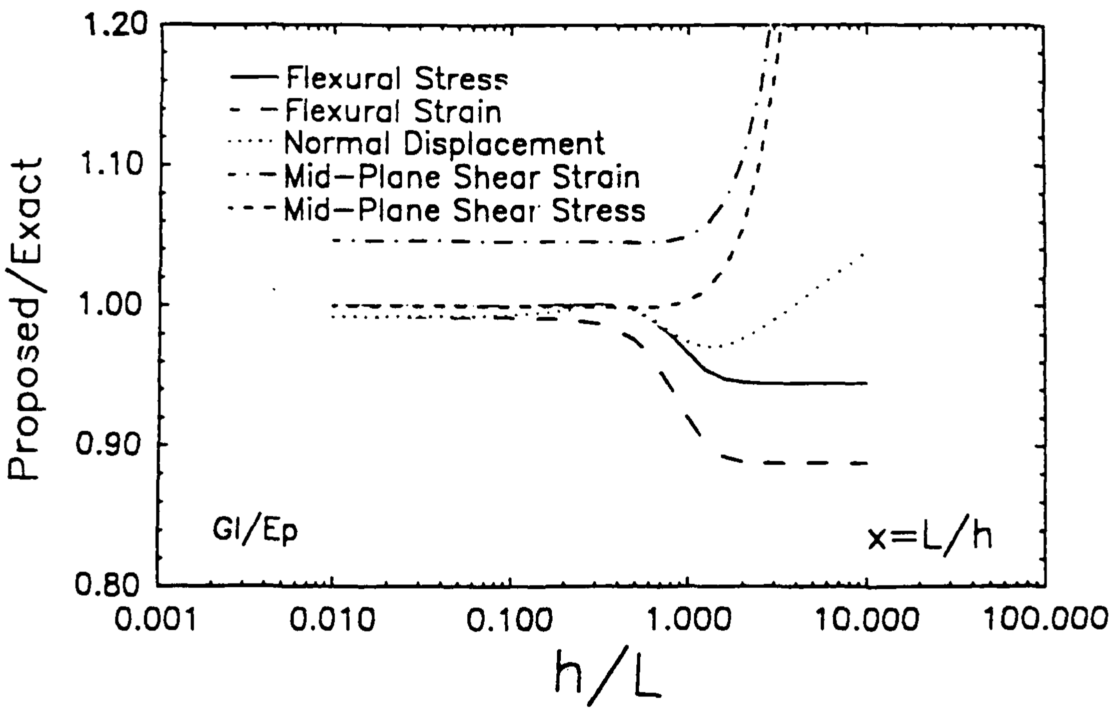

Figure 4. Normalized Field Quantities, as a Function of the Thickness to Loading Wavelength Ratio, Calculated Using the Proposed Constitutive Relationship and Normalized by Their Corresponding Exact Values, for Gl/Ep.

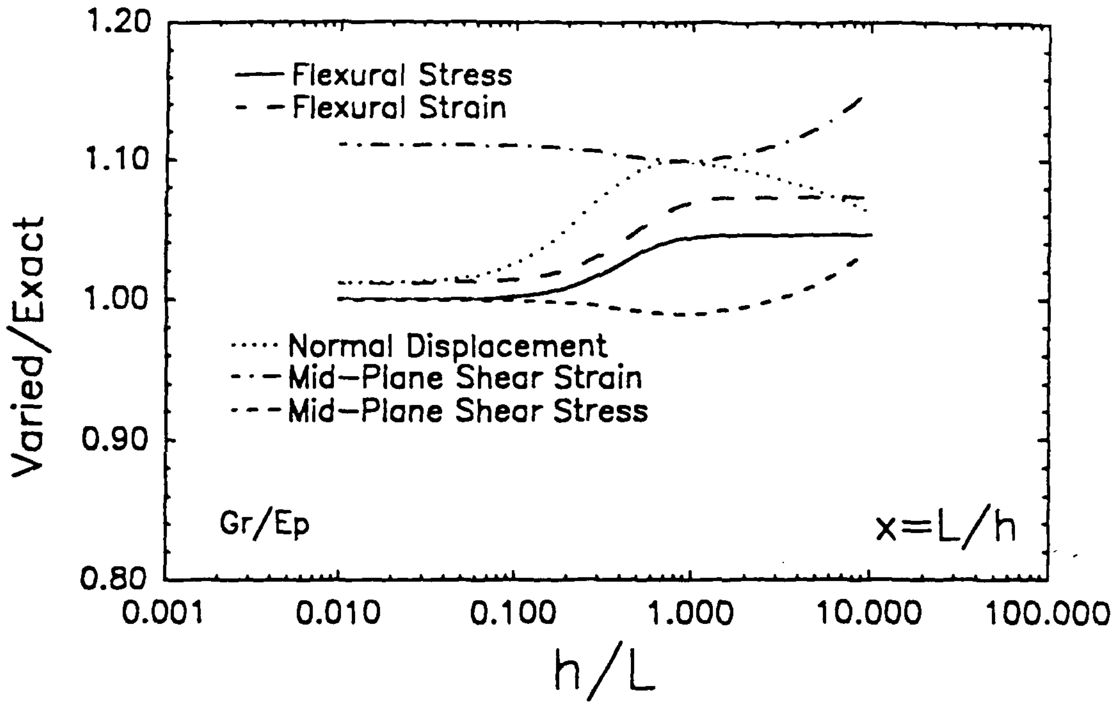

Figure 5. Field Quantities, Normalized by the Exact Solution, vs. the Thickness to Loading Wavelength Ratio. Values Obtained Using the Exact Constitutive Relationship but by Decreasing the Three Matrix-Dominated Properties by 10%, for Gr/Ep.

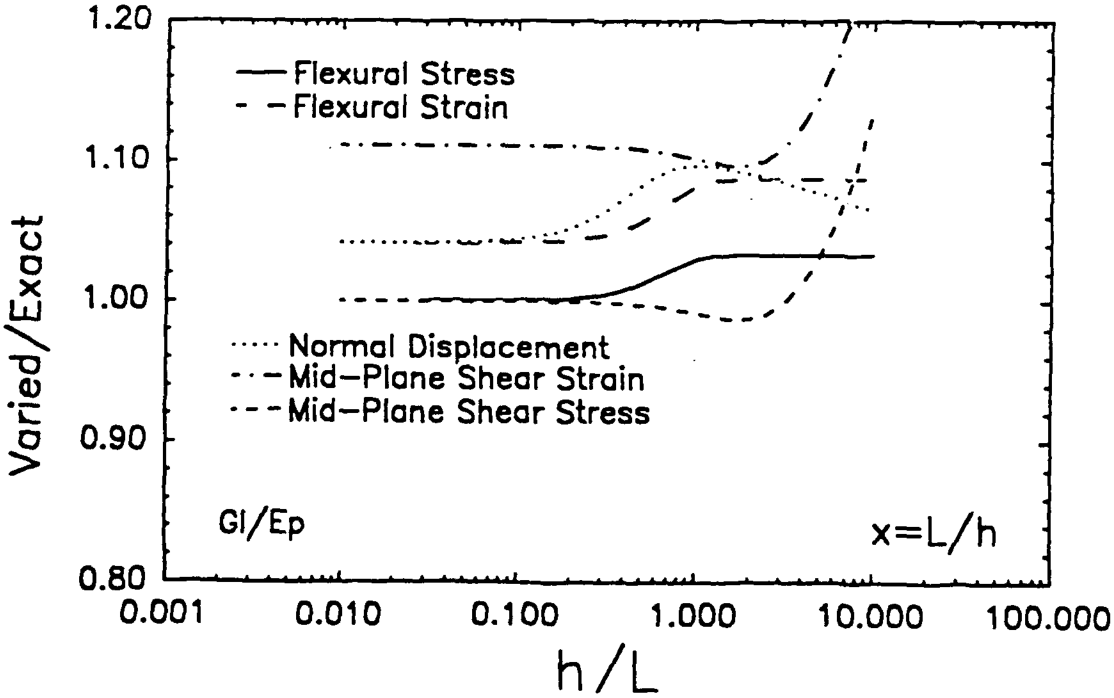

Figure 6. Field Quantities, Normalized by the Exact Solution, vs. the Thickness to Loading Wavelength Ratio. Values Obtained Using the Exact Constitutive Relationship but by Decreasing the Three Matrix Dominated Properties by 10%, for Gl/Ep.

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